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Superfast motion of resistive domains in anisotropic superconductors caused by eddy currents

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Abstract. A new mechanism of superfast motion of resistive domains in anisotropic superconductors is suggested. For bicrystals or sandwiches of anisotropic superconducting materials, the superfast motion is shown to arise due to the additional Joule heating of the kink regions of a resistive domain by eddy currents associated with them. Conditions under which the resistive domain in an anisotropic superconductor moves with a velocity up to the Fermi velocity, $\sim 10^8$ cm s^{-1}, are discussed. As a result of the superfast motion, generation of electric oscillations of high amplitude with frequencies up to 10^{10} Hz can take place.

It has been theoretically and experimentally proved that, under Joule heating, temperature–electric field bistability and resistive (electric) domain instabilities may exist both in superconductors [1–7] and in normal metals [3, 8–12].

In a superconductor the bistability can appear at currents whose magnitudes are less than the critical current but large enough to heat the metal (when it is in the normal state) up to temperatures higher than the critical one. As a result, a resistive domain (RD) (that is, a 'hot spot', which is a normal-metal region of a finite size; the temperature of the 'hot spot' is higher than the critical temperature of the superconductor) spontaneously arises in a long enough bridge of a superconductor. At the same time this domain is a domain of the electric field (the resistive domain is stable under the condition that the voltage applied to the sample is kept fixed) [1–3].

In a normal metal under Joule heating at low temperatures, the bistability appears due to the sharp dependence of the metal resistivity on temperature. In this case the current–voltage characteristic of the metal is N-shaped with a negative differential conductivity associated with the range of temperatures (reached due to the Joule heating) where the electron–phonon scattering dominates over the electron–impurity scattering. At the applied voltage corresponding to this heating, a temperature–electric field domain (TED) spontaneously arises in a long enough metallic wire [3, 8–12].

We note here that in contrast to those in semiconductors, temperature–electric field domains in normal metals and resistive domains in superconductors develop under the condition of local electric neutrality; that is, these domains are inhomogeneous distributions of the temperature and the electric field along the sample length, while the charge remains

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homogeneously distributed (with an accuracy of the order of $\lambda_F/L_0 \ll 1$ where λ_F and L_0 are the Fermi wavelength and the characteristic length in the domain, respectively) providing that there is electric neutrality at every point of the sample; see, e.g., [3].

Under the conditions that the sample and the heat removal are homogeneous, for the case of isotropic thermal and electric conductivities, RDs and TEDs can move only due to the thermoelectric effect [3, 8, 9, 13, 14]. Since the thermoelectric coefficient of a metal (see, e.g., [15]) is proportional to $k_B T/\varepsilon_F \ll 1$ (k_B is Boltzmann's constant and ε_F is the Fermi energy), the thermal domain velocity v is so small ($v \sim 10^{-1}$–10^2 cm s^{-1}) that even weak inhomogeneities lead to pinning of the domain. In [16] an anisotropy in the thermal conductivity of a normal metal was shown to increase the TED velocity by several orders of magnitude. In order to realize this situation for superfast motion of TEDs in normal metals, a specially prepared sample placed in a strong magnetic field was proposed in [16].

However, fast motion of neither resistive domains in superconductors nor temperature–electric field domains in normal metals has been observed in experiments carried out so far.

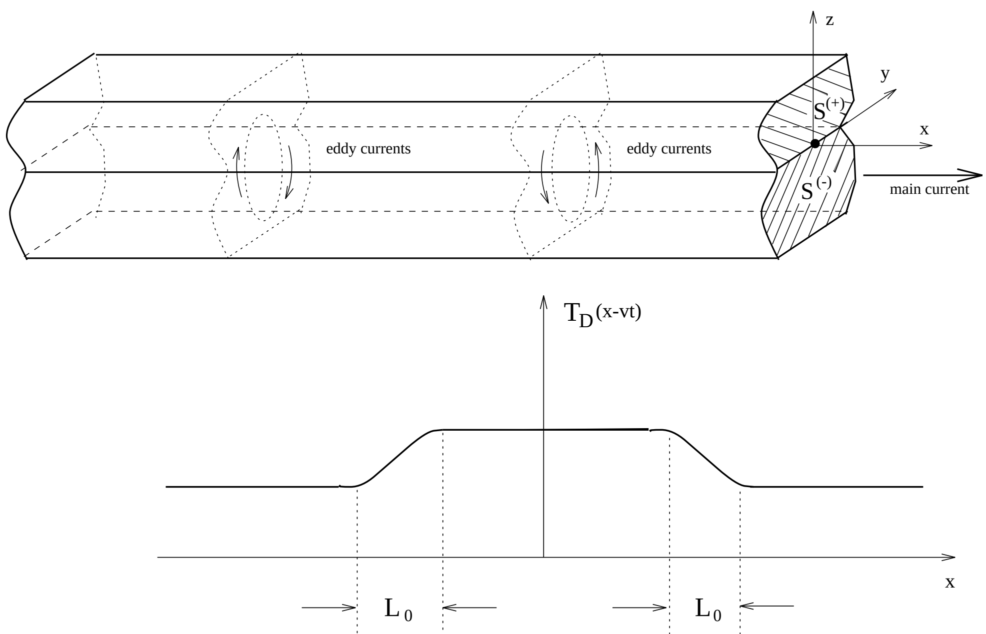

Figure 1. Top: the sandwich composed of two anisotropic metals with the interface boundary along the bridge (the coordinate axis z is perpendicular to the interface boundary; the axes x and y are in its plane with the x-axis directed along the main current direction). Bottom: the inhomogeneous distribution of the temperature in the resistive domain $T_D(x)$.

In the present paper a new mechanism of superfast motion of resistive domains ('hot spots') in superconductors, $T_D(x)$, with anisotropic resistivity is proposed, and conditions under which the resistive domain moves with a velocity close to the Fermi velocity, $\sim 10^8$ cm s^{-1}, are discussed [17]. The basis of the mechanism suggested is an additional Joule heating of the transition regions (kink regions) of a resistive domain caused by eddy currents that arise there due to the inhomogeneous distribution of the temperature in the resistive domain $T_D(x)$ (the coordinate axis x is in the direction of the current along the bridge—see figure 1; these eddy currents arise in a bridge of a metal with anisotropic resistivities $\rho_{ik}(T)$, $i, k = x, y, z$).

As these eddy currents $\vec{j}^{(1)}$ associated with the kink regions should vanish for homogeneous temperature distribution (in this case the resistivity tensor is independent of the coordinates), they are proportional to $\partial T_D/\partial x$; that is, they flow in opposite directions in the left-hand and right-hand kinks of the RDs. On the other hand, the direction of the main current $\vec{j}^{(0)}$ is fixed, and the additional Joule heating of the kink regions by the eddy currents

$$\delta(\vec{j}\cdot\vec{\mathcal{E}}) = (\rho_{xk} + \rho_{kx}) j_x^{(0)} j_k^{(1)} \propto \partial T/\partial x$$

can, in principle, decrease the Joule heating of one kink and increase that of the other ($\vec{\mathcal{E}}$ is the electric field; summation with respect to the double subscript is assumed). If this is the case, one end of the resistive domain is additionally effectively heated and the other is additionally effectively cooled by the eddy currents, and the temperature distribution $T_D(x)$ is shifted to the warmer region; that is, the resistive domain moves together with the eddy currents associated with the kinks [18].

However, for a bridge with a homogeneous resistivity in the cross-section, the additional Joule heating averaged over the cross-sectional area

$$\delta\overline{(\vec{j}\cdot\vec{\mathcal{E}})}$$

(here and below the bar indicates averaging over the cross-section of the bridge) is equal to zero (in the first and the following odd approximations in the $d/L_0 \ll 1$ parameter, d is the ratio between the cross-sectional area and its perimeter, and L_0 is the characteristic length of the kink regions), and hence the eddy currents do not affect the motion of the resistive domain. The situation changes qualitatively if the bridge is a bicrystal or a sandwich of two different anisotropic metals (see figure 1). In this case the additional Joule heatings are different in the uppermost and the lowest layers of such a bridge. As a result the additional Joule heating averaged over the cross-sectional area of the bridge is not equal to zero, being of opposite sign in the left-hand and right-hand kinks (as the heatings are proportional to $\partial T/\partial x$), which causes superfast motion of the resistive domain.

In order to show this, below, the additional Joule heating by the eddy currents is calculated for a long (of length L) superconducting bridge of a sandwich geometry with a resistive domain (that is a 'hot spot') inside it. The sandwich is composed of two anisotropic metals with the interface boundary along the bridge (see figure 1; the coordinate axis z is perpendicular to the interface boundary; the axes x and y are in its plane with the x-axis directed along the main current direction). The metals have different resistivity tensors $\rho_{ik}^{(\pm)}(T)$; superscript '(+)' and superscript '(−)' refer to the uppermost and the lowest layers of the sandwich, respectively. The inequality $d \ll L_0 \ll L$ is assumed to be satisfied. For the sake of simplicity we also assume that

$$\rho_{\alpha x}^{(+)}/\rho_{xx}^{(+)} = \rho_{\alpha x}^{(-)}/\rho_{xx}^{(-)} \equiv \beta_\alpha(T) \qquad (\alpha = y, z)$$

because it eliminates the eddy currents that otherwise appear in the absence of the resistive domain due to the inhomogeneity in the cross-section of the sandwich (these currents are irrelevant to the problem as they do not affect the resistive domain motion).

Under the above assumptions, in the zeroth approximation in $d/L_0 \ll 1$ the Maxwell equations and the boundary conditions are satisfied with the following current and electric field:

$$j_{x,0}^{(\pm)}(T_D(x)) = \frac{\rho_{xx}^{(\mp)}}{S^{(+)}\rho^{(-)} + S^{(-)}\rho^{(+)}} I \qquad \mathcal{E}_{k,0} = \rho_{kx}^{(+)} j_{x,0}^{(+)} = \rho_{xk}^{(-)} j_{x,0}^{(-)} \qquad (1)$$

where I is the total current through the bridge.

In order to find the velocity of the 'hot spot' $T_D(x)$, we need know not the distribution of the eddy currents in the bridge but just that of those averaged over the bridge cross-section, because in the first approximation in $d/L_0 \ll 1$ the velocity is determined by the Joule heating:

$$\delta\overline{\vec{j}\cdot\vec{\mathcal{E}}} \approx \rho_{x\alpha}^{(s)} j_x^{(0)} \overline{j_\alpha^{(1)}}$$

(here $\rho_{x\alpha}^{(s)} \equiv (1/2)(\rho_{x\alpha} + \rho_{\alpha x})$). This permits us to solve the problem for an arbitrary form of the bridge cross-section in the following simple way. We multiply Maxwell's equation for the current by $2\mathcal{E}_{\alpha,0}(\overline{T}(x))\beta_\alpha(\overline{T}(x)$ and integrate it over the sandwich cross-sectional area. Integrating by parts, and taking into account the boundary conditions for the current ($j_\alpha = 0$ on the surface of the bridge and $j_z^{(+)} = j_z^{(-)}$ on the interface boundary $z = 0$), we get the following Joule heating of the sandwich by the eddy currents averaged over the cross-section of the sandwich:

$$\begin{aligned} &\delta\overline{(\vec{j}\cdot\vec{\mathcal{E}})} = \gamma(\partial\overline{T})\frac{\partial\overline{T}}{\partial x} \\ &\gamma(\partial\overline{T}) = \beta_z(\overline{T})\left(\frac{S^{(+)}\rho_{xx}^{(-)}(\overline{T}) - S^{(-)}\rho_{xx}^{(+)}(\overline{T})}{S^{(+)}\rho_{xx}^{(-)}(\overline{T}) + S^{(-)}\rho_{xx}^{(+)}(\overline{T})}\right)'(\langle z\rangle^{(+)} - \langle z\rangle^{(-)})(j_0\mathcal{E}_0). \end{aligned} \tag{2}$$

Here

$$\langle z\rangle^{(\pm)} = (1/S^{(\pm)})\int_{S^{(\pm)}} z\,\mathrm{d}y\,\mathrm{d}z.$$

The prime means the derivative with respect to temperature; $j_0 = I/(S^{(+)} + S^{(-)})$.

The hot-spot velocity v is determined by the condition that the motion of the kinks compensates for the additional heat liberated inside them:

$$c_V^{(0)} T^{(0)} v \sim \delta\overline{(\vec{j}\cdot\vec{\mathcal{E}})} L_0$$

(c_V is the heat capacity of the metal per unit volume; $T^{(0)}$ is the characteristic temperature of the resistive domain). From this and equation (2), it follows that the hot-spot velocity can be estimated as

$$v \sim \frac{\rho_{xx}^{(+)} - \rho_{xx}^{(-)}}{\rho_{xx}^{(0)}}(\rho_{xz}^{(0)} j_x^{(0)2}) d/T^{(0)} c_V^{(0)}. \tag{3}$$

Thus, the resistive domain velocity does not contain small factors of the type $k_B T/\epsilon_F$.

The resistive domain dynamics in a thin sandwich is described by an effective equation for the thermal conductivity for the temperature $\overline{T} = \overline{T}(x,t)$. This equation can be obtained by averaging the three-dimensional equation for the thermal conductivity over the sample cross-section. If one expands the temperature in a power series in d/L_0, and confines consideration to the approximation quadratic in d/L_0, one gets

$$\begin{aligned} &\overline{c_V}(\overline{T})\frac{\partial\overline{T}}{\partial t} - \gamma\frac{\partial\overline{T}}{\partial x} - \frac{\partial}{\partial x}\left(\kappa_{eff}(\overline{T})\frac{\partial\overline{T}}{\partial x}\right) = f(\overline{T}, j_0) \\ &f(\overline{T}, j_0) = j_0\mathcal{E}_0 - \langle q_s(\overline{T})\rangle/d. \end{aligned} \tag{4}$$

Here $\kappa_{eff} = \overline{(1/\lambda_{xx}^{-1})}$ is the effective thermal conductivity, $\lambda_{ik}^{\pm}(T)$ which are present in the cross-sectional average are the thermal resistivity tensors for the upper and lower metals of the sandwich, $q_s(T)$ is the heat flux from a unit surface of the sample, and $\langle\cdots\rangle$ indicates averaging along the contour of the cross-section.

The drift term in the effective thermal conductivity, equation (4), is smaller than the remaining terms by a factor of d/L_0. This leads to the following formula for the velocity v of the resistive domain $\overline{T} = T_D(x - vt)$:

$$v = \int_{T_0}^{T_{\max}}\left[[\gamma(T)\sqrt{W(T, j_c)}\,\mathrm{d}T]\Big/\left(\int_{T_0}^{T_{\max}}\overline{c_V}(T)\sqrt{W(T, j_c)}\,\mathrm{d}T\right)\right] \tag{5}$$

where

$$W = -\int_{T_0}^{T} \kappa_{eff}(T_1) f(T_1, j)\, \mathrm{d}T_1.$$

T_0 is the temperature of the cooling medium, $T_{\max}$ is the temperature of the hot part of the resistive domain; and the current j_c is determined by the equation $W(T_{\max}, j) = 0$. Expressions (2) and (5) give an estimate for the velocity that coincides with equation (3) [19].

From equation (3) and equation (5) it follows that the velocity v of the resistive domain increases as the applied voltage is increased while the maximal temperature $T_{(\max)}$ is kept as low as possible. It can be reached if the external heat removal q_s is improved. However, increase of the heat removal is limited by the maximal possible heat flux inside the sample. According to the heat current conservation law, an increase of the external heat-removing flux q_s (together with an increase of the Joule heating) leads to an increase of the transverse heat flux $q_z^{(i)}$ inside the sample. But the maximal possible value of $q_z^{(i)}$ is

$$q_{(\max)}^{(i)} \sim \kappa_0^{(i)} T / l_0^{(i)}$$

($\kappa_0^{(i)}$ and $l_0^{(i)}$ are the characteristic values of the thermal conductivity and the heat-carrier free path length inside the metal of the sample), and therefore from equation (3) and the estimate of the maximal possible Joule heating $(j_0\mathcal{E}_0)d \sim q_{(\max)}^{(i))}$, it follows that the maximal possible velocity of the thermal domains is

$$v^{(\max)} \sim (\kappa_0^{(i)} T / l_0^{(i)}) / (c_V^{(i)} T) \simeq v^{(i)} \tag{6}$$

($c_V^{(i)}$ is the heat capacity of the sample metal, and $v^{(i)}$ is the velocity of the heat carriers in the sample). Therefore the maximal velocity is obtained if the heat removal through the external medium q_s reaches $q_{(\max)}^{(i)}$. This can be achieved if two conditions are satisfied: (1) the heat-removal medium is a metal whose thickness $d^{(e)}$ is less than or of the order of the free path length of the electrons inside the external metal $l_0^{(e)}$, and (2) the maximal temperature $T_{\max}$ in the resistive domain is close to liquid helium temperature. Hence in bridges of superconducting anisotropic conventional metals, resistive domains can move at velocities of the order of the Fermi velocity, $v_F \sim 10^8$ cm s^{-1}. For highly anisotropic HTSC material with a critical temperature T_c close to helium temperatures, the maximal velocity of the resistive domain is of the order of the sound velocity, $s \sim 10^5$ cm s^{-1}, if T_c is in the range of temperatures for which the thermal conductivity of the HTSC material is determined by phonons; if the thermal conductivity is determined by electrons, the maximal resistive domain velocity is of the order of the characteristic velocity of electrons of the HTSC material (see equation (6)).

It seems that an experimental observation of the superfast motion of resistive domains in superconductors could be of a great interest, as it can lead to the generation of electric oscillations of high amplitude and of high frequency, up to 10^{10} Hz.

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References

[1] Volkov A F and Kogan Sh M 1974 *JETP Lett.* **19** 4
[2] Skocpol W I, Beasley M R and Tinkham M 1974 *J. Appl. Phys.* **45** 4054
[3] Gurevich A Vl and Mints R G 1987 *Rev. Mod. Phys.* **59** 941
[4] Poulin G D, Hegmann F A, Lachapelle J, Moffat S H and Preston J S 1995 *IEEE Trans. Appl. Supercond.* **5**

[5] Morgoon V N, Jardim R F, Bindilatti V, Becerra C C, Rodrigues D Jr, Bondarenko A V and Sivakov V 1996 *J. Supercond.* **9** 129
[6] Okamoto T, Huybrechts B and Takata M 1994 *Japan. J. Appl. Phys.* **33** L1212
[7] Barelko V V and Zhukov S A 1995 *High Temp.* **33** 68
[8] Slutskin A A and Kadigrobov A M 1978 *JETP Lett.* **28** 201
[9] Kadigrobov A M, Slutskin A A and Krivoshei I V 1984 *Sov. Phys.–JETP* **60** 754
[10] Boiko V V, Podrezov Yu F and Klimova N P 1982 *Pis. Zh. Eksp. Teor. Fiz.* **35** 524 (Engl. Transl. 1982 *JETP Lett.* **35** 649)
[11] Chiang Yu N and Logvinov I I 1982 *Fiz. Nizk. Temp.* **8** 774 (Engl. Transl. 1982 *Sov. J. Low Temp. Phys.* **8** 388)
[12] Abramov G I, Gurevich A V, Dzugutov V M, Mints R G and Fisher L M 1983 *Pis. Zh. Eksp. Teor. Fiz.* **37** 453 (Engl. Transl. 1983 *JETP Lett.* **37** 535)
[13] Gurevich A Vl and Mints R G 1980 *Pis. Zh. Eksp. Teor. Fiz.* **31** 52 (Engl. Transl. 1980 *JETP Lett.* **31** 48)
[14] Ausloos M 1981 *Physica* B **108** 969
[15] Landau L D, and Lifshitz E M 1982 *Electrodinamika Sploshnykh Sred (Electrodynamics of Continuous Media)*
[16] Kadigrobov A M and Slutskin A A 1994 *Low Temp. Phys.* **20** 224
[17] As is mentioned above, the resistive domain is a soliton-type distribution of the temperature and the electric field along the length of the sample while the density of electrons and the current along the sample remain constant. As a result, this motion of the domain does not cause irradiation of the Cherenkov type, and hence the radiation friction decreasing the velocity is absent.
[18] In this case the eddy currents manifest themselves as an effective thermoelectric Thomson effect in the RD motion but they do not contain small factors of the type $k_B T/\epsilon_F$.
[19] The mutual transformation of longitudinal and transverse heat fluxes in the sample considered in [16] also leads to motion of the resistive domain but it does not change this estimate of the resistive domain velocity. On the other hand the superfast-motion mechanism suggested in this paper works for TEDs in normal metals as well, but in order to achieve a low temperature of the hot part of the TED (which is necessary for high velocities) one has to construct the sample and the cooling media in the way described in [16].